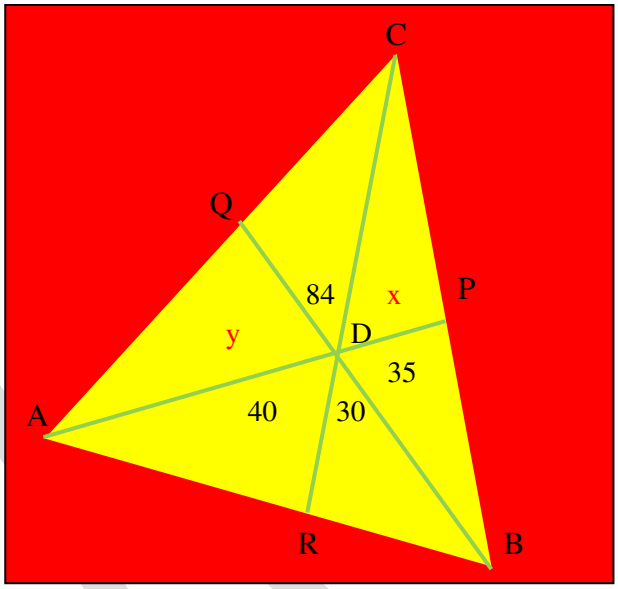
# Properties of Triangle

Q) In ΔABC, lines through the vertices A, B and C intersect in a common point D. The areas of different triangles in sq. units are as indicated in the diagram.
Find the area of ΔABC.




Solution:

Let AD intersect BC in P, BD intersect CA in Q and CD intersect AB in R.
Let the area of ΔCDP be 'x' and that of ΔADQ be 'y' respectively.

We make use of the following property:

For triangles having bases on the same line and having a common vertex, their areas are in the ratio of their bases.

⇨ $\frac{x}{35} = \frac{CP}{BP}$ .....(i) Bases CP and BP on the same line BC with D as common vertex

⇨ $\frac{x+84+y}{35+30+40} = \frac{CP}{BP}$

⇨ $\frac{x+y+84}{105} = \frac{CP}{BP}$...(ii) Bases CP and BP on the same line BC and A is common vertex

From (i) and (ii), $\frac{x+y+84}{105} = \frac{x}{35}$

⇨ $y = 2x - 84$ ……(iii)

⇨ Again, $\frac{y}{84} = \frac{AQ}{CQ}$ ....(iv) Bases AQ and CQ on the same line CA and D is common vertex

⇨ $\frac{y+40+30}{84+x+35} = \frac{AQ}{CQ}$

⇨ $\frac{y+70}{x+119} = \frac{AQ}{CQ}$ …(v) Bases AQ and CQ on the same line CA and B is common vertex

From (iv) and (v) $\frac{y+70}{x+119} = \frac{y}{84}$

⇨ $35y + xy = 84\ x\ 70$ …… (vi)

From (iii) and (vi) $35y + y\left(\frac{y+84}{2}\right) = 84x70$

⇨ $y^2 + 154y - (2\ x\ 70\ x\ 84) = 0$

⇨ $(y + 210)(y - 56) = 0$

⇨ $y = 56$

⇨ $x = 70$ From (iii)

⇨ $Area\ of\ \Delta ABC = 70 + 35 + 30 + 40 + 56 + 84 = 315\ sq.units$