Circles and Triangle
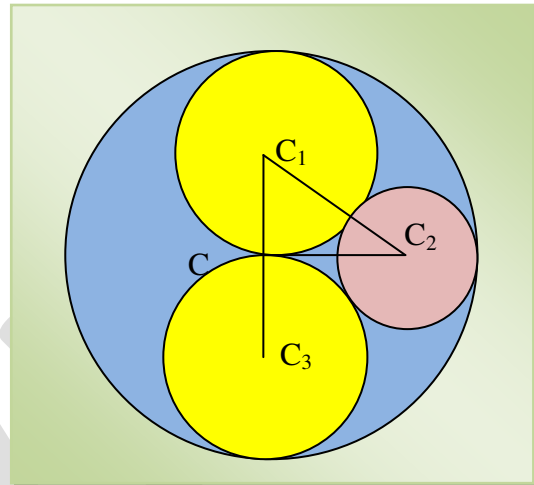
Q) The diagram shows a circle with radius 24, which contains two circles with radius 12 and tangent to each other and the larger circle.
The smallest circle is tangent to the three other circles.
What is the radius of the smallest circle?

Solution:

Since the circles are tangent to each other,
Circles with centres C_1 and C_3 (radius 12 units) pass through Centre C of the outermost circle.(radius 24 units).

Also, circle with centre C_2 is tangent to the other three circles.
$\Rightarrow CC_2$ is $\perp$ to C_1C_3.

Let the radius of the smallest circle be *'r'*.

Applying Pythagoras Theorem

$$(C_1C_2)^2 = (CC_1)^2 + (CC_2)^2$$
$$\Rightarrow (r+12)^2 = (12)^2 + (24-r)^2$$
$$\Rightarrow r^2 + 24r + (12)^2 = (12)^2 + (24)^2 - 48r + r^2$$
$$\Rightarrow 72r = 576$$
$$\Rightarrow r = 8 \; units$$