Binomial Theorem: Properties of Binomial Coefficients

Q) If $S_n = \sum_{r=0}^{r=n} \frac{1}{{}^nC_r}$ *and* $t_n = \sum_{r=0}^{r=n} \frac{r}{{}^nC_r}$, where ${}^nC_r$ represents the Binomial Coefficient,

then $\frac{t_n}{S_n} =$ ___________

(a) $\frac{n}{2}$ (b) $\frac{n}{2}-1$ (c) $n-1$ (d) $\frac{2n-1}{2}$

Solution: Correct option (a)

$$S_n = \sum_{r=0}^{r=n} \frac{1}{{}^nC_r}$$

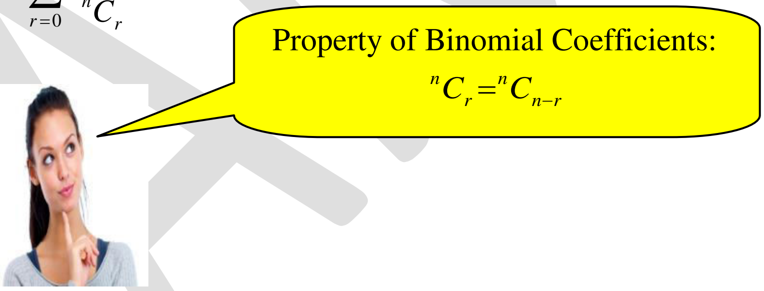
$$= \frac{1}{n}\sum_{r=0}^{r=n} \frac{n}{{}^nC_r}$$

$$= \frac{1}{n}\sum_{r=0}^{r=n} \frac{(n-r)+r}{{}^nC_r}$$

$$\Rightarrow nS_n = \sum_{r=0}^{r=n} \frac{n-r}{{}^nC_r} + \sum_{r=0}^{r=n} \frac{r}{{}^nC_r}$$

$$= \sum_{r=0}^{r=n} \frac{n-r}{{}^nC_r} + t_n \qquad \Lambda\,(i)$$

Consider $\sum_{r=0}^{r=n} \frac{n-r}{{}^nC_r}$



$$\Rightarrow \sum_{r=0}^{r=n} \frac{n-r}{{}^nC_r} = \sum_{r=0}^{r=n} \frac{n-r}{{}^nC_{n-r}}$$

Replace $n-r=m$

$$r=0 \Rightarrow m=n \quad and \quad r=n \Rightarrow m=o$$

Then $\sum_{r=0}^{r=n} \frac{n-r}{{}^nC_{n-r}} = \sum_{m=n}^{m=0} \frac{m}{{}^nC_m}$

$$= \sum_{m=0}^{m=n} \frac{m}{{}^nC_m}$$

$$= \sum_{r=0}^{r=n} \frac{r}{{}^nC_r} = t_n \qquad \Lambda\,(ii)$$

From (i) and (ii)

$$nS_n = t_n + t_n = 2t_n$$

$$\Rightarrow \frac{t_n}{S_n} = \frac{n}{2}$$