Binomial Theorem

Q) Prove that, for a positive integer n,

$$11^{n+2}+12^{2n+1} \text{ is divisible by } 133$$

Solution:

Concept used: Binomial Expansion

$$(a+b)^n = {}^nC_{\circ}a^nb^{\circ} + {}^nC_1a^{n-1}b^1 + {}^nC_2a^{n-2}b^2 + \ldots + {}^nC_ra^{n-r}b^r + \cdots + {}^nC_{n-1}a^1b^{n-1} + {}^nC_na^{\circ}b^n$$

$$\begin{aligned}
&11^{n+2}+12^{2n+1}\\
&=121(11)^n+12(144)^n\\
&=121(11)^n+12(133+11)^n\\
&=121(11)^n+12\left[{}^nC_0(133)^n+{}^nC_1(133)^{n-1}(11)^1+\cdots+{}^nC_{n-1}(133)^1(11)^{n-1}+{}^nC_n(11)^n\right]\\
&=12\left[{}^nC_0(133)^n+{}^nC_1(133)^{n-1}(11)^1+\cdots+{}^nC_{n-1}(133)^1(11)^{n-1}\right]+\left[121(11)^n+12\,{}^nC_n(11)^n\right]\\
&=12\times133\left[{}^nC_0(133)^{n-1}+{}^nC_1(133)^{n-2}(11)^1+\cdots+{}^nC_{n-1}(11)^{n-1}\right]+(11)^n(121+12)\\
&=12\times133\left[{}^nC_0(133)^{n-1}+{}^nC_1(133)^{n-2}(11)^1+\cdots+{}^nC_{n-1}(11)^{n-1}\right]+133(11)^n\\
&\Rightarrow \textit{Divisible by } 133
\end{aligned}$$